

# A Plethora of Negative-Refractive Phenomenons in Relativistic and Non-Relativistic Scenarios

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## Abstract

In accordance with Snel’s law of refraction, whether a plane wave is refracted in the negative sense or positive sense at a planar boundary between two homogenous mediums is determined solely by the orientation of the real parts of the wavevectors involved. Thus, negative refraction should be distinguished from the associated but independent phenomenons of negative phase velocity, counterposition and negative deflection of energy flux. None of these phenomenons is Lorentz covariant.

## 1 Introduction

Despite optical refraction being one of the oldest topics of scientific investigation, interest in the phenomenon of negative refraction has been widespread only for the past ten years. This interest stemmed from experimental reports of negative refraction, at microwave frequencies, in certain carefully engineered metamaterials [1]. More recently, negative refraction in metamaterials has been reported at higher frequencies, with the visible frequency range now almost attained [2]. Research on negative refraction has been largely fuelled by the prospect of manufacturing planar lenses from negatively refracting materials [3]. While these lenses would not be ‘perfect’ [4, 5], they may have a very high resolving power.

The present-day scientific literature on optics abounds with descriptions of purported negative refraction in a variety of complex scenarios. However, some inconsistencies are apparent between reports over what exactly constitutes negative refraction. Sometimes what is actually being described is not negative refraction *per se*, but an associated phenomenon such as negative phase velocity, counterposition or negative deflection of energy flux [6, 7]. While these phenomenons can—and often do—arise in conjunction with negative refraction, it is important to bear in mind that these are independent phenomenons which should be distinguished from negative refraction, especially so in complex materials and in relativistic scenarios.

We highlight here the distinctions between negative refraction and the associated phenomenons of negative phase velocity, counterposition and negative deflection of energy flux. These distinctions are apparent in both relativistic and non-relativistic scenarios.

## 2 Negative refraction

Suppose a plane wave propagates in medium I with wavevector  $\mathbf{k}_I$ , towards a planar interface separating medium I and medium II, such that  $\text{Re}\{\mathbf{k}_I\} \cdot \hat{\mathbf{n}} > 0$ , where  $\hat{\mathbf{n}}$  is the unit vector normal to the interface

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directed into medium II. In keeping with Snel's law of refraction [8], the plane wave is said to be negatively refracted at the planar interface between mediums I and II, if the dot product  $\text{Re}\{\mathbf{k}_{II}\} \cdot \hat{\mathbf{n}} < 0$  where  $\mathbf{k}_{II}$  is the wave vector in medium II. Conversely, if  $\text{Re}\{\mathbf{k}_{II}\} \cdot \hat{\mathbf{n}} > 0$  then the plane wave is said to be positively refracted. A schematic illustration is provided in Fig. 1.

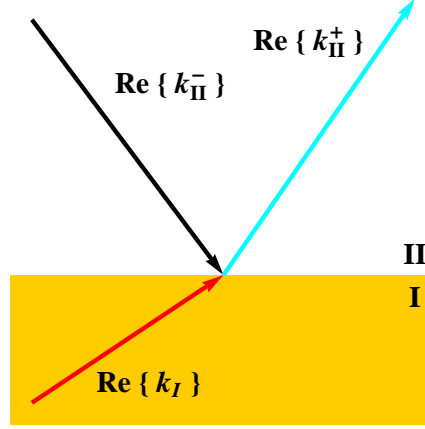


Figure 1: A schematic illustration of refraction of a plane wave at the planar interface between mediums I and II. Suppose a uniform plane wave propagates in medium I with wavevector  $\mathbf{k}_I$ , towards the interface. If it is negatively refracted at the interface then its wavevector in medium II, namely  $\mathbf{k}_{II}^-$ , is such that  $\text{Re}\{\mathbf{k}_{II}^-\} \cdot \hat{\mathbf{n}} < 0$ , where  $\hat{\mathbf{n}}$  is the unit vector normal to the interface directed into medium II. If it is positively refracted at the interface then its wavevector in medium II, namely  $\mathbf{k}_{II}^+$ , is such that  $\text{Re}\{\mathbf{k}_{II}^+\} \cdot \hat{\mathbf{n}} > 0$ .

In the case of uniform plane waves propagating in passive isotropic dielectric–magnetic materials, negative refraction is synonymous with negative phase velocity. However, the correspondence between negative refraction and negative phase velocity breaks down if the materials are active [9], or if nonuniform plane waves are considered [7], or if anisotropic or bianisotropic materials are involved [7].

Furthermore, whether a plane wave is refracted positively or negatively, depends upon the inertial frame of reference of the observer, as was recently demonstrated using a pseudochiral omega material moving at uniform velocity [10].

### 3 Negative phase velocity

The sign associated with a plane wave's velocity refers to the direction of the phase velocity vector relative to the corresponding time-averaged Poynting vector. A plane wave with phase velocity  $\mathbf{v}$  and time-averaged Poynting vector  $\mathbf{P}$  has negative phase velocity if  $\mathbf{v} \cdot \mathbf{P} < 0$  and positive phase velocity if  $\mathbf{v} \cdot \mathbf{P} > 0$ . We note that orthogonal phase velocity, i.e.,  $\mathbf{v} \cdot \mathbf{P} = 0$ , is also a possibility [7]. Although negative phase velocity and negative refraction often accompany each other, they need not. The schematic Fig. 2 shows how negative phase velocity can arise with or without negative refraction.

While the simplest material that can support negative refraction is an isotropic dielectric–magnetic material, it is notable that even an isotropic dielectric material can support negative phase velocity (in conjunction with positive refraction) [7]. However, there is greater scope for negative phase velocity in more complex materials such as isotropic chiral [11, 12] and bianisotropic [13, 14] materials.

As is the case for refraction, the sign of a plane wave's phase velocity is not Lorentz covariant [15].<sup>3</sup> A noncovariant formalism—wherein vacuum in curved spacetime is formally represented by a fictitious nonhomogeneous bianisotropic medium in flat spacetime [17]—has been widely used to investigate the prospects of

<sup>3</sup>A covariant analogue of the NPV condition  $\mathbf{k} \cdot \mathbf{P} < 0$  has been derived, where  $\mathbf{k}$  is a real-valued wavevector [16]. However, the physical basis for this is unclear because there is no *a priori* reason for the quantity  $\mathbf{k} \cdot \mathbf{P}$  to be covariant.

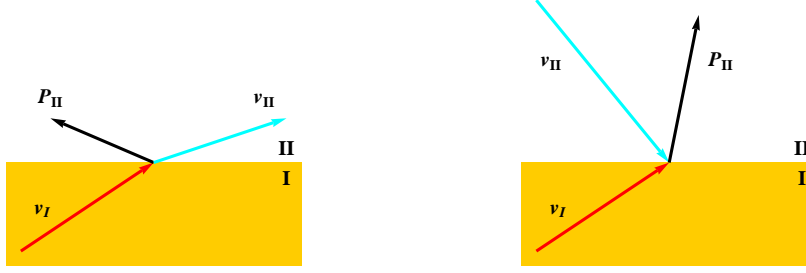


Figure 2: Two schematic examples of negative phase velocity, arising at the planar interface between mediums I and II. Suppose a plane wave propagates in medium I with phase velocity  $\mathbf{v}_I$ , towards the interface. The corresponding plane wave in medium II has negative phase velocity if  $\mathbf{v}_{II} \cdot \mathbf{P}_{II} < 0$ , where  $\mathbf{v}_{II}$  and  $\mathbf{P}_{II}$  are the phase velocity and time-averaged Poynting vector, respectively, of the plane wave in medium II. In the example on the left side, the plane wave in medium II is positively refracted and the wavevector and time-averaged Poynting vector are counterposed. In the example on the right side, the plane wave in medium II is negatively refracted and the wavevector and time-averaged Poynting vector are not counterposed.

negative phase velocity in vacuum under the influence of strong gravitational fields [18, 19, 20]. The metrics associated with de Sitter [21], Kerr [23, 24], Kerr–Newman [25], Reissner–Nordström [26], Schwarzschild [27], and Schwarzschild–de Sitter [28] spacetimes have been found to support negative phase velocity. Let us note that negative phase velocity in such general relativistic scenarios is quite distinct from superradiance, although both phenomena involve negative energy densities [29].

## 4 Counterposition

Consider a plane wave, propagating in medium I towards a planar interface between mediums I and II. The real part of the wavevector and the time-averaged Poynting vector of the resulting plane wave which is launched in medium II are said to be counterposed if these two vectors lie on opposite sides of the unit vector normal to the interface between mediums I and II. A schematic illustration of two different manifestations of counterposition—one arising in conjunction with negative refraction and the other not—is provided in Fig 3.

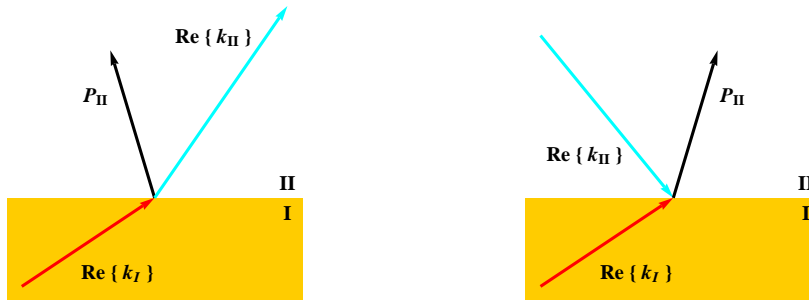


Figure 3: Two schematic examples of counterposition, arising at the planar interface between mediums I and II. Suppose a plane wave propagates in medium I with wavevector  $\mathbf{k}_I$ , towards the interface. The corresponding real part of the wavevector  $\mathbf{k}_{II}$  and time-averaged Poynting vector  $\mathbf{P}_{II}$  in medium II are counterposed if they lie on opposite sides of the the unit vector normal to the interface directed into medium II. In the example on the left side, the plane wave in medium II is positively refracted and has positive phase velocity. In the example on the right side, the plane wave in medium II is negatively refracted and has negative phase velocity.

Counterposition cannot occur in isotropic dielectric–magnetic materials (at rest), but it can in anisotropic materials [30] where it is sometimes referred to as amphoteric refraction [31]. Indeed, counterposition has also been referred to as negative refraction [32]. Similarly to negative refraction and negative phase velocity, counterposition can be induced by motion at constant velocity in materials which do not support counterposition at rest [15, 32, 33].

## 5 Negative deflection of energy flux

Although there is no equivalent of Snel’s law for energy flux, the notion of negative deflection of energy flux has been adopted by some in the negative refraction community [31, 34, 35, 36, 37]. The energy flux associated with a plane wave—as represented by the time–averaged Poynting vector—is negatively deflected at a planar boundary between mediums I and II if the time–averaged Poynting vectors in mediums I and II lie on opposite sides of the unit vector normal to the interface. A schematic illustration of negative and positive energy flux deflection is provided in Fig. 4. For uniform plane waves in isotropic dielectric–magnetic materials, the direction of energy flux is parallel or anti–parallel to the wavevector, but for more complex materials and/or nonuniform plane waves this is not the case [7]. Thus, the energy flux can be deflected negatively even if though the plane wave is refracted positively, and vice versa. Furthermore, whether or not the energy flux is deflected negatively depends upon the inertial reference frame of the observer [15].

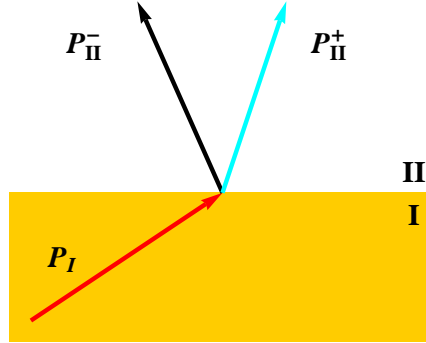


Figure 4: A schematic illustration of deflection of a energy flux of a plane wave, at the planar interface between mediums I and II. Suppose a plane wave propagates in medium I with time–averaged Poynting vector  $\mathbf{P}_I$ , towards the interface, such that  $\mathbf{P}_I \cdot \hat{\mathbf{t}} > 0$  where  $\hat{\mathbf{t}}$  is a unit vector lying along the interface between mediums I and II, in the plane of incidence. If the plane wave’s energy flux is negatively deflected at the interface then the corresponding time–averaged Poynting vector in medium II, namely  $\mathbf{P}_{II}^-$ , is such that  $\mathbf{P}_{II}^- \cdot \hat{\mathbf{t}} < 0$ . If it is positively deflected at the interface then the corresponding time–averaged Poynting vector in medium II, namely  $\mathbf{P}_{II}^+$ , is such that  $\mathbf{P}_{II}^+ \cdot \hat{\mathbf{t}} > 0$ .

From a practical point of view, the direction of energy flow associated with a beam is likely to be of greater significance than the energy flow associated with a plane wave. Of course, many standard optical devices can be straightforwardly implemented to deflect beams in any direction, without the need for complex materials or metamaterials, and it is also possible to suppress beam deflection [38, 39]. A recent theoretical study involving beam propagation through uniformly moving slabs has demonstrated that whether a beam is negatively or positively deflected depends upon the motion of the slab relative to an observer [15]. This result could be harnessed to achieve a degree of concealment: if a slab moves at a certain specific velocity relative to an observer, a beam can propagate through the slab with no deflection at all [40].

## 6 Closing remarks

Negative refraction is already the focus of considerable research efforts and it may well play significant roles in future optical technologies. It is therefore important to unambiguously establish exactly what is negative refraction, and to distinguish it from the associated but independent phenomena of negative phase velocity, counterposition and negative deflection of energy flux. These distinctions are of particular importance when more complex materials and metamaterials are considered, and in relativistic scenarios.

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<sup>4</sup>Report NASA TT-F477 is the English translation of a book published originally in Russian in 1951.